GARCH Analysis

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$$x_t = \mu + a_t$$ $$\sigma_t^2 = \alpha_o + \sum_{i=1}^p {\alpha_i a_{t-i}^2}+\sum_{j=1}^q{\beta_j \sigma_{t-j}^2}$$ $$a_t = \sigma_t \times \epsilon_t$$ $$\epsilon_t \sim P_{\nu}(0,1)$$ Where:

- x_t is the time series value at time t.
- μ is the mean of GARCH in Excel model.
- a_t is the model's residual at time t.
- σ_t is the conditional standard deviation (i.e. volatility) at time t.
- p is the order of the ARCH component model.
- $\alpha_o,\alpha_1,\alpha_2,...,\alpha_p$ are the parameters of the the ARCH component model.
- q is the order of the GARCH component model.
- $\beta_1,\beta_2,...,\beta_q$ are the parameters of the the GARCH component model.
- $\left[\epsilon_t\right]$ are the standardized residuals: $$\left[\epsilon_t\right] \sim i.i.d$$ $$E\left[\epsilon_t\right]=0$$ $$\mathit{VAR}\left[\epsilon_t\right]=1$$
- P_{ν} is the probability distribution function for ϵ_t. Currently, the following distributions are supported:
  1. Normal distribution $P_{\nu} = N(0,1)$.
  2. Student's t-distribution $P_{\nu} = t_{\nu}(0,1)$ $\nu \succ 4$
  3. Generalized error distribution (GED) $$P_{\nu} = \mathit{GED}_{\nu}(0,1)$$ $$\nu \succ 1$$

- **Clustering**: a large a_{t-1}^2 or σ_{t-1}^2 gives rise to a large σ_t^2. This means a large a_{t-1}^2 tends to be followed by another large a_{t}^2, generating, the well-known behavior, of volatility clustering in financial time series.
- **Fat-tails**: The tail distribution of a GARCH in Excel (p,q) process is heavier than that of a normal distribution.
- **Mean-reversion**: GARCH in Excel provides a simple parametric function that can be used to describe the volatility evolution. The model converge to the unconditional variance of a_t: $$\sigma_{\infty}^2 \rightarrow V_L=\frac{\alpha_o}{1-\sum_{i=1}^{max(p,q)}\left(\alpha_i+\beta_i\right)}$$

See Also

[template("related")]