SARIMAX

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In principle, an SARIMAX model is a linear regression model that uses a SARIMA-type process (i.e. w_t) This model is useful in cases we suspect that residuals may exhibit a seasonal trend or pattern.

$$ w_t = y_t - \beta_1 x_{1,t}-\beta_2 x_{2,t} - \cdots - \beta_b x_{b,t}$$
$$ (1-\sum_{i=1}^p {\phi_i L^i})(1-\sum_{j=1}^P {\Phi_j L^{j \times s}})(1-L)^d (1-L^s)^D w_t -\eta = (1+\sum_{i=1}^q {\theta_i L^i})(1+\sum_{j=1}^Q {\Theta_j L^{j \times s}}) a_t $$
$$ a_t \sim \textrm{i.i.d} \sim \Phi(0,\sigma^2) $$

Where:

- L is the lag (aka back-shift) operator.
- y_t is the observed output at time t.
- $x_{k,t}$ is the k-th exogenous input variable at time t.
- β_k is the coefficient value for the k-th exogenous (explanatory) input variable.
- b is the number of exogenous input variables.
- w_t is the auto-correlated regression residuals.
- p is the order of the non-seasonal AR component.
- P is the order of the seasonal AR component.
- q is the order of the non-seasonal MA component.
- Q is the order of the seasonal MA component.
- s is the seasonal length.
- D is the seasonal integration order of the time series.
- η is a constant in the SARIMA model
- a_t is the innovation, shock or error term at time t.
- $\{a_t\}$ time series observations are independent and identically distributed (i.e. i.i.d) and follow a Gaussian distribution (i.e. $\Phi(0,\sigma^2)$)

Re-ordering the terms in the equation above and assuming the differenced (both seasonal and non-seasonal) results in a stationary time series (z_t) yields the following:

$$ z_t = (1-L)^d(1-L^s)^D w_t$$
$$ \mu = E[z_t] = \frac{\eta}{(1-\phi_1-\phi_2-\cdots-\phi_p)(1-\Phi_1-\Phi_2-\cdots-\Phi_P)}$$
$$ (1-\sum_{i=1}^p {\phi_i L^i})(1-\sum_{j=1}^P {\Phi_j L^{j \times s}})(1-L)^d (1-L^s)^D (w_t -\mu) = (1+\sum_{i=1}^q {\theta_i L^i})(1+\sum_{j=1}^Q {\Theta_j L^{j \times s}}) a_t$$

Remarks

1. The variance of the shocks is constant or time-invariant.
2. The order of an AR component process is solely determined by the order of the last lagged auto-regressive variable with a non-zero coefficient (i.e. w_{t-p}).
3. The order of an MA component process is solely determined by the order of the last moving average variable with a non-zero coefficient (i.e. a_{t-q}).
4. In principle, you can have fewer parameters than the orders of the model.

Requirements

Header	SFSDK.H
Library	SFSDK.LIB
DLL	SFSDK.DLL

References

Hamilton, J .D.; Time Series Analysis , Princeton University Press (1994), ISBN 0-691-04289-6

Tsay, Ruey S.; Analysis of Financial Time Series John Wiley & SONS. (2005), ISBN 0-471-690740

See Also

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